X-bar and R Charts

X-bar Charts & R Charts

- Used for measurement data
- Assumes population is normally distributed
- Upper and lower control limits usually 3 standard deviations above and below the mean of the process

Constructing x-bar charts

- Collect small samples in equally spaced intervals over time
- Usually 20-30 periods
- n is the sample size collected each time
- k is the number of samples collected over time

Constructing x-bar charts

- For each sample, calculate the mean

$$\bar{x} = \frac{\sum x_i}{n}$$

- For each sample, calculate the range

$$range = x_{largest} - x_{smallest}$$

Constructing x-bar charts

- Calculate the grand or overall mean by averaging all the sample means

$$\bar{\bar{x}} = \frac{\sum \bar{x}_i}{k}$$

This becomes the center line of the x-bar chart.

Constructing x-bar charts

- Calculate the r-bar or overall mean of the ranges next

$$\bar{R} = \frac{\sum r_i}{k}$$

This is needed to calculate the upper and lower control limits of the x-bar chart.

Constructing x-bar charts

- Calculate the upper and lower control limits

$$ControlLimits_{\bar{x}} = \bar{\bar{x}} \pm A_2 \bar{R}$$

- To find A_2, one must use tables for control chart factors

Constructing x-bar charts

- On the next slide, find a chart of factors for both x-bar and r charts
- To find an appropriate factor, look for the sample size, n, collected each time

Constructing x-bar charts

- So if one had chosen 4 items to measure each time, use 0.729 for A_2.
- D_3 and D_4 are used in calculating upper and lower control limits for r charts.

n	A2	D3	D4
2	1.88	0	3.267
3	1.023	0	2.575
4	0.729	0	2.282
5	0.577	0	2.115
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

Constructing r charts

- The center line for the r chart is already calculated with the calculation of r-bar
- Now just calculate the upper and lower control limits

Constructing r charts

- Calculate the upper control limit

$$UCL_R = D_4 \overline{R}$$

- Calculate the lower control limit

$$LCL_R = D_3 \overline{R}$$

An Example

- A bank manager decides to study waiting times of customers for teller service at the noon lunch hour (noon to 1pm). He selects one customer every 20 minutes to measure the time the customer enters the line to the time he or she reaches the teller window. He records this over several days.

Example data

Day	Time in Minutes		
1	7.2	8.4	7.9
2	5.6	8.7	3.3
3	5.5	7.3	3.2
4	4.4	8.0	5.4
5	9.7	4.6	4.8
6	8.3	8.9	9.1

Calculated x-bar and r

Day	Time in Minutes			xbar	r
1	7.2	8.4	7.9	7.83	1.2
2	5.6	8.7	3.3	5.87	5.4
3	5.5	7.3	3.2	5.33	4.1
4	4.4	8.0	5.4	5.93	3.6
5	9.7	4.6	4.8	6.37	5.1
6	8.3	8.9	9.1	8.77	0.8

Calculate center lines

$$\bar{\bar{x}} = \frac{\sum \bar{x}_i}{k} \qquad \bar{R} = \frac{\sum r_i}{k}$$

$= 40.1/6$ $= 20.2/6$

$= 6.68$ $= 3.37$

Calculate control limits

$ControlLimits_{\bar{x}} = \bar{\bar{x}} \pm A_2 \bar{R}$ = 6.68 ± 1.023(3.37)

= 6.68 ± 3.45

Upper Limit = 10.13

Lower Limit = 3.23

$UCL_R = D_4 \bar{R}$ = 2.575(3.37)

= 8.68

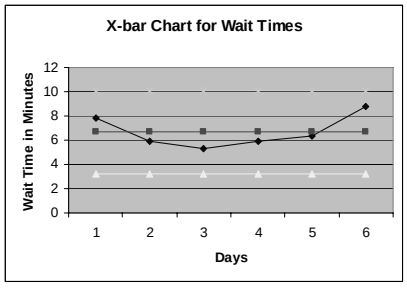
$LCL_R = D_3 \bar{R}$ = 0(3.37)

= 0

Drawing the x-bar chart

- Draw the center line first
- Place the lower and upper control limits on the graph
- Plot the means for each sample
- Connect the points

Drawing the x-bar chart



Drawing the r chart

- Process is the same for all charts!
- Draw the center line first
- Place the lower and upper control limits on the graph
- Plot the ranges for each sample
- Connect the points

Drawing the r chart